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UNIFIED COUNCIL OLYMPIADS

UIMO
Unified International Mathematics Olympiad

UNIFIED INTERNATIONAL MATHEMATICS OLYMPIAD

CLASS - 9
Question Paper Code : UM9274

KEY

1	2	3	4	5	6	7	8	9	10
C	A	D	D	A	A	A	A	B	C
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
D	B	A	D	A	A	A	A	A	D
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
C	D	C	D	C	C	B	D	B	D
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
A,B,C,D	B,C	A,B,C,D	B,C	B,C	C	C	C	B	B
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
A	D	D	D	B	C	C	D	B	D

EXPLANATIONS

MATHEMATICS - 1 (MCQ)

1. (C) $2^{4\left(x^2+3x-1\right)}=2^{3\left(x^2+3x+2\right)}$

$4x^2 + 12x - 4 = 3x^2 + 9x + 6$

$\Rightarrow x^2 + 3x - 10 = 0$

or $(x + 5)(x - 2) = 0$

$\therefore\ x = -5, 2$

Sum of all values of "x" = –5 + 2 = –3

2. (A) AD = AO + OD = $\dfrac{AE}{2}+\dfrac{AO}{2}$

= 14 cm + 7 cm

= 21 cm

OC = OD = 14 cm

Area of the shaded region = Area of sector

Area of the parallelogram = Area of sector

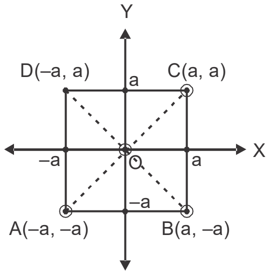
AOC – Area of ΔCOD

$= 21 \times 14\ cm^2 - 90° \times \frac{22}{7} \times 14 \times 14\ cm^2$

$-\frac{1}{2} \times 14 \times 7\ cm^2$

$= 294\ cm^2 - 154\ cm^2 - 49\ cm^2 = 91\ cm^2$

3. (D) Given points are A(–a, –a), B(a, –a), C (a, a) and D(–a, a)



Hence, it is clear that the given points form a square and the origin lies at the point where the diagonals of the square intersect.

4. (D) Given that the radii of three solid glass balls are 'r' cm, 6 cm and 8 cm, sum of the volumes of the three glass balls

$= \frac{4}{3}\pi r^3 + \frac{4}{3}\pi(6)^3 + \frac{4}{3}\pi(8)^3$

$= \frac{4}{3}\pi\,(r^3 + 6^3 + 8^3)\ cm^3$

The volume of the solid sphere of radius 9 cm

$= \frac{4}{3}\pi(9^3) = 243 \times 4\pi$

$\therefore\ 243 \times 4\pi = \frac{4}{3}\pi\ (r^3 + 728)$

$\Rightarrow 729 = r^3 + 728$

$\Rightarrow r^3 = 729 - 728 = 1$

$\Rightarrow r = 1$

Hence, r = 1 cm

5. (A) Semicircular arc BC = 6π

$\Rightarrow$ Circumference of circle with diameter BC = $2 \times 6\pi = 12\pi$

$\Rightarrow$ Diameter = 12 = Side BC of rectangle ABCD.

Similarly, length of semicircular arc CD = 4π

$\Rightarrow$ Its diameter = 8 = side CD of rectangle ABCD

Therefore, area of rectangle

ABCD = BC $\times$ CD = 12×8 = 96 Sq. units

6. (A) Let $p(x) = x^4 - a^2\,{}^2 + 3x - a$.

Since $x + a$, i.e. $x - (-a)$ is a factor of $p(x)$, we must have $p(-a) = 0$

$\Rightarrow (-a)^4 - a^2(-a)^2 + 3(-a) - a = 0$

$\Rightarrow a^4 - a^4 - 3a - a = 0$

$\Rightarrow -4a = 0$

$\Rightarrow a = 0$

7. (A) Let the two consecutive even numbers be 'n' and (n + 2).

Then, according to the problem,

$n^2 + (n + 2)^2 = 340$

$\Rightarrow n^2 + n^2 + 4n + 4 = 340$

$\Rightarrow 2n^2 + 4n + 4 = 340$

$\Rightarrow 2n^2 + 4n - 336 = 0$

$\Rightarrow n^2 + 2n - 168 = 0$

$\Rightarrow n^2 + 14n - 12n - 168 = 0$

$\Rightarrow n(n + 14) - 12(n + 14) = 0$

$\Rightarrow (n + 14)(n - 12) = 0$

$\Rightarrow n = -14$ or 12

$\therefore$ The required numbers are 12 and 14

Their sum = 12 + 14 = 26.

8. (A) Total cost for painting

$= [2h(l + b)] \times$ ₹ 4

$= 12 \times 15 \times 4$

= ₹ 720

9. (B) Given $(x^2 - 3x + 2)$ is a factor of

$p(x) = x^4 - px^2 + q$

$x^2 - 3x + 2 = (x - 1)(x - 2)$

$(x - 1)$ is a factor of $p(x)$

$1 - p + q = 0$

$p - q = 1$

$p = q + 1$ .... (1)

$(x - 2)$ is also a factor of $p(x)$

$2^4 - p(2)^2 + q = 0$

$16 - 4p + q = 0$

$16 - 4(q + 1) + q = 0$

$16 - 4q - 4 + q = 0$

$12 - 3q = 0$

$12 = 3q \Rightarrow q = 4$

$p = q + 1 = 4 + 1 = 5$

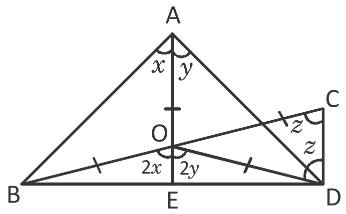
10. (C) LHS = $\sqrt[3]{\left(\sqrt[3]{x}\right)^3+3\left(\sqrt[3]{x}\right)^2 3y+3\sqrt[3]{x}\sqrt[3]{y}+\left(\sqrt[3]{y}\right)^3}$

$=\sqrt[3]{\left(\sqrt[3]{x}+\sqrt[3]{y}\right)^3}$

$=\left(\sqrt[3]{x}+\sqrt[3]{y}\right)^{3\times\frac{1}{3}}$

$= \sqrt[3]{x}+\sqrt[3]{y}$

11. (D)



'O' is equidistant from A, B, C and D

∴ 'O' is the centre of the circle

∴ $\angle BCD$

$= \angle BAD = 70°$

[∴ Angle in the semicircle]

(OR)

'O' is circumcentre of ΔABC

$\angle BAD$ & $\angle BAC$ are angles in the same segment $\Rightarrow$ $\angle BCD = \angle BAD = 70°$

Const:- External AO up to E (or)

In ΔAOB, given OA = OB

$\Rightarrow \angle OBA = \angle OAB = x$

In ΔAOD given OA = OD

$\Rightarrow \angle ODA = \angle OAD = y$

$\therefore \angle BOE = x + x = 2x$

$\angle DOE = y + y = 2y$

$\therefore \angle BOD = \angle BOE + \angle DOE$

$= 2x + 2y = 2(x + y) = 2 \times 70°$

$= 140°$

$\therefore \angle DOC = 180° - \angle BOD = 40°$

In ΔCOD, $OC = OD \Rightarrow \angle ODC = \angle OCD = z$

In ΔCOD, $z + z + 40° = 180°$

$z = 70°$

$\therefore \angle BCD = z = 70°$

12. (B) Volume of shades solid

$= 4 \times 6 \times 5 - 1 \times 2 \times 4 = 112$ units^3

13. (A) Mass = V × D = $\pi(R + r)(R - r)h \times D$

$= \frac{22}{7}\left(\frac{4.5}{2}+2\right)\left(\frac{4.5}{2}-2\right)77 \times 8$ gm/cc

= 2.057 kg

14. (D) $\angle PQR = 90^o$ [∴ Angle in a semi circle]

$\therefore \angle QPR + \angle QRP = 90^o$

$\angle QPR + 30^o = 90^o$

$\angle QPR = 60^o$

$\therefore \angle TPR = 100^o - 60^o = 40^o$

But $\angle TPR + \angle x = 180^o$

$40^o + x = 18^o$

$x = 140^o$

15. (A) In ΔABC, $\angle B = 90^o = AC^2 = AB^2 + BC^2$

$41^2 = AB^2 + 40^2$

AB = 9

Area of

$\Delta ABC = \frac{1}{2} \times AB \times BC$

$= \frac{1}{2} \times 9 \times 40cm^2 = 180cm^2$

In DABC, $\angle ACD = 90^o$ is $= AB^2 = AC^2 + CB^2$

$841^2 = 41^2 + CB^2$

CD = 840

Area of

$\Delta ACD = \frac{1}{2} \times AC \times CD$

$= \frac{1}{2} \times$ 41cm × 840cm

$= 17{,}220cm^2$

Total area = 17,220 cm^2 + 180cm^2
= 17,400 cm^2

16. (A) (x -1) is a factor means sum of coefficient are zero.

17. (A) PXQY is a parallelogram

18. (A) In ΔADC, $\angle D = 90^o$

∴ $AB^2 = AD^2 + DB^2$

$(15cm)^2 = (9cm)^2 + DB^2$

$225cm^2 - 81cm^2 = DB^2$

$DC = \sqrt{144\ cm^2} = 12$ cm

19. (A)

20. (D) Given $4\pi r^2 = 1018\frac{2}{7} cm^2$

$$4 \times \frac{22}{7} \times r^2 = \frac{7128}{7} cm^2$$

$$\therefore\ r^2 = \frac{\cancel{7128}^{\cancel{648}\ \cancel{324}\ 81}}{\cancel{7}} cm^2 \times \frac{\cancel{7}}{\cancel{22}_{2}} \times \frac{1}{\cancel{4}_{1}}$$

$r^2 = (9cm)^2$

r = 9 cm

Volume of sphere $= \frac{4}{3}\pi r^3$

$$= \frac{4}{\cancel{3}} \times \frac{22}{7} \times \cancel{9}^{3} \times 9 \times 9\ cm^3$$

$= 3054.85\ cm^3$

$= 3054.9\ cm^3$

21. (C) $s = \frac{a+b+c}{2} = \frac{9cm + 40cm + 41cm}{2} = \frac{90cm}{2} = 45cm$

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{45cm \times 36cm \times 5cm \times 4cm}$$

$$= \sqrt{9 \times 5 \times 9 \times 4 \times 5 \times 4cm^4}$$

$= 9 \times 5 \times 4\ cm^2 = 180\ cm^2$

$\therefore \frac{1}{2} \times 9cm \times h = 180cm^2$

[∵ Shortest side altitude is longest]

$$h = 180\ cm^2 \times \frac{2}{9cm} = 40cm$$

22. (D) Degree of $(x^2 + 1)^3$ is 6

Degree of $(x^3 + 1)^4$ is 12

∴ Degree of $(x^2 + 1)^3 (x^3 + 1)^4 = 6 + 12 = 18$

23. (C)

$$x - \frac{1}{x}\ \Bigg|\ \begin{array}{l} x^2 + \frac{1}{x^2} \\ x^2 - 1 \\ (-)\ \ (+) \\ \hline 1 + \frac{1}{x^2} \end{array}\ \Bigg|\ x$$

24. (D) $\sqrt{448} - \sqrt{1008} - \sqrt{567} + \sqrt{700}$

$= \sqrt{64 \times 7} - \sqrt{144 \times 7} - \sqrt{81 \times 7} + \sqrt{100 \times 7}$

$= 8\sqrt{7} - 12\sqrt{70} - 9\sqrt{7} + 10\sqrt{7}$

$= -3\sqrt{7}$

$= -\sqrt{3 \times 3 \times 7}$

$= -\sqrt{63}$

25. (C) $x^2 + x(c - b) + (c - a)(a - b) = x^2 + x(c - a + a - b) + (c - a)(a - b)$

$= x^2 + x[(c - a) + (a - b)] + (c - a)(c - b)$

$= x^2 + x(c - a) + x(a - b) + (c - a)(a - b)$

$= x(x + c - a) + (a - b)(x + c - a)$

$= (x + c - a)(x + a - b)$

26. (C) In $\Delta PSQ'$, $PQ' = 2PS$ & $\angle PSQ' = 90°$

∴ $(PQ')^2 = PS^2 + (SQ')^2$

$(2PS)^2 = PS^2 + (SQ')^2$

$(SQ')^2 = 3(PS^2)$

$SQ' = \sqrt{3(PS)^2} = \sqrt{3}\ PS$

In $\Delta PSQ'$, the sides ratio

$= PS : \sqrt{3} PS : 2 PS$

$= 1 : \sqrt{3} : 2$

Angles ratio = 1 : 2 : 3 = 30° : 60° : 90°

$\angle SPQ' = 60°$ $\angle Q'PQ = 30°$

$\angle Q'PX = \frac{\angle Q'PQ}{2} = 15°$

$\angle SP = 60° + 15° = 75°$

27. (B) Given $x + \frac{1}{x} = 5.2 = 5 + 0.2 = 5 + \frac{1}{5}$

$$\therefore x = 5 \Rightarrow x^3 + \frac{1}{x^3} = 5^3 + \frac{1}{5^3}$$

$$= 125 + \frac{1}{125} = 125.008$$

(OR)

Given $x+\frac{1}{x}=\frac{52}{10}=\frac{26}{5}$

Cubing on both sides

$$\left(x+\frac{1}{x}\right)^3=5.2^3$$

$$x^3+\frac{1}{x^3}+3x\times\frac{1}{x}\left(x+\frac{1}{x}\right)=140.608$$

$$x^3+\frac{1}{x^3}+3(5.2)=140.608$$

$$x^3+\frac{1}{x^3}=140.608-15.6=125.008$$

28. (D) Const:- Join BD

In ΔBCD given BC = CD

$\angle$BDC = $\angle$CBD = a

In ΔBCD a + a + 50° = 180°

2a = 180° – 50° = 130°

$$a=\frac{130^\circ}{2}=65^\circ$$

$\therefore$ In a cyclic quadrilateral ABDE, BDC = x

$\therefore$ x = $\angle$BCDB = 65°

29. (B) $\frac{\sqrt[6]{36}}{\sqrt[3]{3}}=\frac{\sqrt[6]{36}}{\sqrt[6]{3^2}}=\sqrt[6]{\frac{\cancel{36}^{4}}{\cancel{9}}}=\sqrt[6]{4}=\sqrt[6]{2^2}=\sqrt[3]{2}$

30. (D) In the circle having centre A, 0

we have AC = AB. ...... (1)

(Since each is equal to the radius of the circle)

In the circle having centre B, we have BC = AB. ...... (2)

(Since each is equal to the radius of the circle)

$\therefore$ From (1) and (2), we have AB = BC = AC
Hence, ΔABC is equilateral.

MATHEMATICS - 2 (MAQ)

31. (A,B,C,D) Let $\left(5\sqrt{2},-3\sqrt{3}\right)$ lies on $\sqrt{3}x+\sqrt{2}y$

LHS = $\sqrt{3}\times5\sqrt{2}+\sqrt{2}\times\left(-3\sqrt{3}\right)$

$=5\sqrt{6}-3\sqrt{6}$

$=2\sqrt{6}=$ R.H.S

Similarly $\left(0,\sqrt{12}\right),\left(\sqrt{8},0\right)$ and $\left(\sqrt{2},\sqrt{3}\right)$ also lie on

$\sqrt{3}x+\sqrt{2}y=2\sqrt{6}$

32. (B,C) A sphere has no flats surface.

33. (A,B,C,D) If ‘n’ (x – 1) is a factor of

p(x) = x^n – 1

p(1) = 0

i.e., 1^n – 1 = 0, when ‘n’ is a natural number, whole number, integer and prime number.

34. (B,C) $3(x + 2)^2 + 2(x + 2)^2 = 48 + 32$

$5(x + 2)^2 = 80$

$(x + 2)^2 = \frac{80}{5}=16$

$x+2=\pm\sqrt{16}$

x + 2 = $\pm$4

x + 2 = 4 or x + 2 = –4

x = 2   x = – 6

35. (B, C) $\angle B=\angle A-9^\circ$; $\angle C=\angle A-72^\circ$

But $\angle A+\angle B+\angle C=180^\circ$

$\angle A+\angle A-9^\circ+\angle A-72^\circ=180^\circ$

$3\angle A=180^\circ+81^\circ$

$\angle A=\frac{261^\circ}{3}=87^\circ$

$\angle B=\angle A-9^\circ=78^\circ$

$\angle C=\angle A-72^\circ$

$\angle C=87^\circ-72^\circ=15^\circ$

REASONING

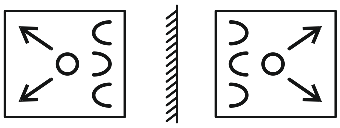
36. (C) From the option 3rd, we get:

$\Rightarrow$ $10 + 10 \div 10 - 10 \times 10 = 10$

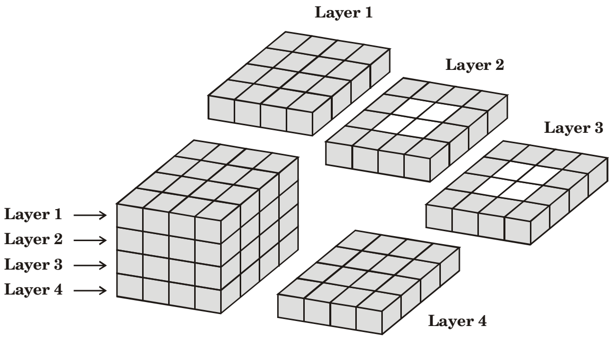
$\Rightarrow$ $10 \times 10 \div 10 - 10 + 10 = 10$

$\Rightarrow$ $10 - 10 + 10 = 10.$

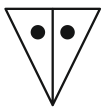
Hence, the option C is correct.

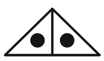
37. (C) 

38. (C) The four squares each of the two layers in between i.e., 8 cubes have no face coloured.



39. (B) From the first sentence it is clear that A is brother of K. Hence option (B) is not true.

40. (B) First letter represents = 

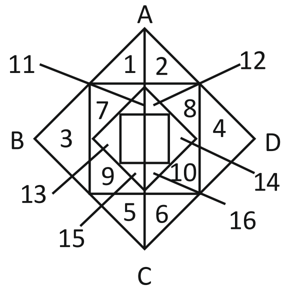
Second letter represents upper part = 

41. (A) Change the Roman numberals into modern numbers;

208 104 (CIV) 52 26 (XXVI)

Each one is half the previous number, therefore the next number is 13, expressed in modern numerals to conform with the established pattern.

42. (D) No. of individual triangles = 16



No. of triangles formed by combinations

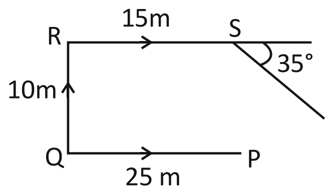
= 1 + 2, 11 + 12, 15 + 16, 5 + 6,

11 + 13 + 15, 12 + 14 + 16,

ABC and ACD

$\therefore$ Total number of triangles = 16 + 8 = 24

43. (D) Cubes of consecutive numbers 1009 is not a cube of 10.

44. (D) 

Hence he should go in south east direction.

45. (B) The fill changes from white to lattice. The sides of the enclosed shape double in number. The shape is enclosed by a circle with a grey fill.

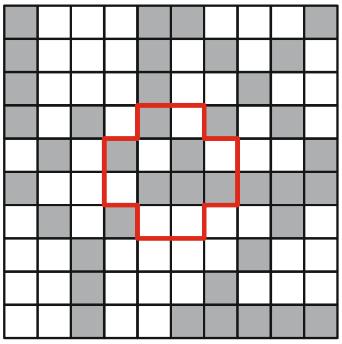
CRITICAL THINKING

46. (C) Potential energy is slowly converted into Kinetic energy during the free fall of an objects.

After it has falen at energy get equally distributed.

47. (C) Each of the squares moves anti-clockwise, first one position, then two, then three and so on.

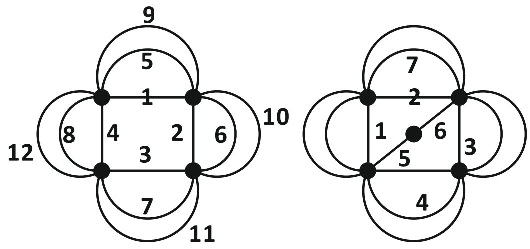
48. (D) Splitting the diagram in half both horizontally and vertically, each quarter contains a pattern of black squares, representing the letters W, X, Y and Z.



49. (B) Only argument II is strong.

For the all-round progress of the nation, all the students, especial the talented and intelligent ones, must avail of higher education, even if the government has to pay for it. So, only argument II holds.

50. (D) In the below image, I had numbered the order of drawing continuous lines (you can have another order also)



(iii) (iv)

The End